

# Cosmology and CP Violation <sup>1</sup>

*Gustavo C.Branco*

Departamento de Fisica and Grupo Teorico de Fisica de Particulas  
Instituto Superior Tecnico  
Av. Rovisco Pais,1049-001 Lisboa, Portugal

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We describe the role of CP violation in the generation of the baryon asymmetry of the Universe, in the framework of baryogenesis through leptogenesis, with emphasis on the possible relationship between CP violation at low energies and that required by leptogenesis. It is emphasized that a direct link between these two manifestations of CP violation only exists in the framework of specific flavour structures for the fundamental leptonic mass matrices.

## 1 Introduction

The phenomenon of CP violation has profound implications for Cosmology, since it is one of the necessary ingredients [1] for generating the observed baryon asymmetry of the Universe (BAU). During the last few years, the data collected from the acoustic peaks in the cosmic microwave background radiation [2] has allowed to obtain a precise measurement of BAU. The MAP experiment [3] and the PLANCK satellite [4] planned for the near future should further improve this result. At the present time, the measurement of the baryon-to-entropy ratio  $Y_B = n_B/s$  is

$$0.7 \times 10^{-10} \lesssim Y_B \lesssim 1.0 \times 10^{-10}. \quad (1)$$

A great challenge for Particle Physics is finding a plausible mechanism capable of reproducing this ratio. Although there are various possible scenarios for baryogenesis one of the most appealing ones is that provided by leptogenesis [5], where first the out-of-equilibrium decay of righthanded neutrinos creates a lepton asymmetry which is then converted into a baryon asymmetry through B-violating but (B-L) conserving sphaleron mediated processes [6]. In any baryogenesis scenario, a fascinating question which naturally arises is whether low energy data on CP violation obtained from terrestrial experiments could give us information on the processes responsible for the creation of BAU. More specifically, in the context of leptogenesis, one may wonder whether it is possible to relate CP violation necessary to generate BAU, to leptonic CP violation at low energies [7], [8], [9], observable through neutrino oscillations. It has been shown that this connection exists only in specific models [8], [9].

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<sup>1</sup>Based on work done in collaboration with R. González Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy in [7]

## 2 A Minimal Extension of the Standard Model

In order to understand the relationship between CP breaking at low energies and CP violation responsible for leptogenesis, one has to specify the particle physics framework one is considering. We will work within a minimal extension of the Standard Model (SM) which consists of adding to the standard spectrum, one right-handed neutrino per generation. At this stage, no other assumption is being made, so our framework is quite general and it is indeed the simplest extension of the SM capable of generating non vanishing but naturally small neutrino masses. Before gauge symmetry breaking, the leptonic couplings to the SM Higgs doublet  $\phi$  can be written as:

$$\mathcal{L}_Y = -Y_\nu (\bar{\ell}_L^0, \bar{\nu}_L^0) \tilde{\phi} \nu_R^0 - Y_\ell (\bar{\ell}_L^0, \bar{\nu}_L^0) \phi \ell_R^0 + \text{H.c.}, \quad (2)$$

where  $\tilde{\phi} = i\tau_2 \phi^*$ . After spontaneous gauge symmetry breaking, the leptonic mass terms are given by:

$$\begin{aligned} \mathcal{L}_m &= - \left[ \bar{\nu}_L^0 m_D \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M_R \nu_R^0 + \bar{\ell}_L^0 m_\ell \ell_R^0 \right] + \text{H.c.} \\ &= - \left[ \frac{1}{2} n_L^T C \mathcal{M}^* n_L + \bar{\ell}_L^0 m_\ell \ell_R^0 \right] + \text{H.c.}, \end{aligned} \quad (3)$$

where  $m_D = v Y_\nu$  is the Dirac neutrino mass matrix with  $v = \langle \phi^0 \rangle / \sqrt{2} \simeq 174 \text{ GeV}$ ,  $M_R$  and  $m_\ell = v Y_\ell$  denote the right-handed Majorana neutrino and charged lepton mass matrices, respectively, and  $n_L = (\nu_L^0, (\nu_R^0)^c)$ . Among all the terms, only the right-handed neutrino Majorana mass term is  $\text{SU}(2) \times \text{U}(1)$  invariant and, as a result, the typical scale of  $M_R$  can be much above the electroweak symmetry breaking scale  $v$ , thus leading to naturally small left-handed Majorana neutrino masses of the order  $m_D^2/M_R$  through the seesaw mechanism. In terms of weak-basis eigenstates the leptonic charged current interactions are given by:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_L^0 \gamma^\mu \nu_L^0 + \text{H.c.} \quad (4)$$

It is clear from Eqs. (3) and (4) that it is possible to choose, without loss of generality, a weak basis (WB) where both  $m_\ell$  and  $M_R$  are diagonal, real and positive. Note that in this WB,  $m_D$  is a general complex matrix which contains all the information on CP-violating phases as well as on leptonic mixing. Since we are considering a standard Higgs sector, in the present framework there is no  $\Delta L = 2$  mass term of the form  $\frac{1}{2} \nu_L^{0T} C M_L \nu_L^0$  at tree level. The total number of CP-violating phases for  $n$  generations is then given by  $n(n-1)$  [10] since one can eliminate  $n$  of the initial  $n^2$  phases of  $m_D$ . All CP violating phases are contained in  $m_D$  in this special weak basis<sup>2</sup>.

In the physical basis (i.e. the mass eigenstates basis) all CP violating phases are shifted to the leptonic mixing matrix appearing in charged weak currents. We recall that the full  $6 \times 6$  neutrino mass matrix  $\mathcal{M}$  is diagonalized via the transformation:

$$V^T \mathcal{M}^* V = \mathcal{D}, \quad (5)$$

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<sup>2</sup>The counting of independent CP-violating phases for the general case, where besides  $m_D$  and  $M_R$  there is also a left-handed Majorana mass term at tree level has been discussed in Ref. [11].

where  $\mathcal{D} = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$ , with  $m_i$  and  $M_i$  denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write  $V$  and  $\mathcal{D}$  in the following form, together with the definition of  $\mathcal{M}$  :

$$V = \begin{pmatrix} K & Q \\ S & T \end{pmatrix} , \quad \mathcal{D} = \begin{pmatrix} d_\nu & 0 \\ 0 & D_R \end{pmatrix} , \quad \mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} . \quad (6)$$

From Eq. (5) one obtains, to an excellent approximation, the seesaw formula:

$$d_\nu \simeq -K^\dagger m_D M_R^{-1} m_D^T K^* \equiv K^\dagger \mathcal{M}_\nu K^* , \quad (7)$$

where  $\mathcal{M}_\nu$  is the usual light neutrino effective mass matrix. The leptonic charged-current interactions are given by:

$$-\frac{g}{\sqrt{2}} \left( \bar{\ell}_L \gamma_\mu K \nu_L + \bar{\ell}_L \gamma_\mu Q N_L \right) W^\mu + \text{H.c.} , \quad (8)$$

where  $\nu_i$  and  $N_i$  denote the light and heavy neutrino mass eigenstates, respectively. The matrix  $K$  which contains all information on mixing and CP violation at low energies can then be parametrized, after eliminating the unphysical phases, by  $K = U_\delta P$  with  $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$  ( $\alpha$  and  $\beta$  are Majorana phases) and  $U_\delta$  a unitary matrix which contains only one (Dirac-type) phase  $\delta$ . In the limit where the heavy neutrinos exactly decouple from the theory, the matrix  $K$  is usually referred as the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix, which from now on we shall denote as  $U_\nu$ . It is clear from Eq.(7) that in general and without further assumptions the phases  $\alpha$ ,  $\beta$  and  $\delta$  are complicated functions of the six independent phases appearing in  $m_D$  ( we are considering 3 generations ) in the weak-basis where  $m_l$  and  $M_R$  are diagonal, real and positive. As we will see in the sequel, this is the essential reason why in general and without further assumptions, it is not possible to establish a direct connection between the phases appearing at low energies and those relevant for leptogenesis.

### 3 CP Violation in Neutrino Oscillations

It has been shown [8] that the strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy WB invariant:

$$\mathcal{T}_{CP} = \text{Tr} [ \mathcal{H}_\nu, H_\ell ]^3 = 6 i \Delta_{21} \Delta_{32} \Delta_{31} \text{Im} [ (\mathcal{H}_\nu)_{12} (\mathcal{H}_\nu)_{23} (\mathcal{H}_\nu)_{31} ] , \quad (9)$$

where  $\mathcal{H}_\nu = \mathcal{M}_\nu \mathcal{M}_\nu^\dagger$ ,  $H_\ell = m_\ell m_\ell^\dagger$  and  $\Delta_{21} = (m_\mu^2 - m_e^2)$  with analogous expressions for  $\Delta_{31}$ ,  $\Delta_{32}$ . This relation can be computed in any weak basis. This is specially useful since most of the ansatzes for the leptonic mass matrices are written in a WB where neither  $\mathcal{H}_\nu$  nor  $H_\ell$  are diagonal. The above WB invariant enables one to investigate whether a specific ansatz leads to CP violation in neutrino oscillations or not, without performing any diagonalization of leptonic mass matrices and without computing  $U_\nu$ . The low-energy invariant (9) is sensitive to the Dirac-type phase  $\delta$  and vanishes for  $\delta = 0$ . On the other hand, it does not depend on the Majorana phases  $\alpha$  and  $\beta$  appearing in the leptonic mixing matrix. The quantity  $\mathcal{T}_{CP}$  can be fully written in terms of physical observables since

$$\text{Im} [ (\mathcal{H}_\nu)_{12} (\mathcal{H}_\nu)_{23} (\mathcal{H}_\nu)_{31} ] = -\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 \mathcal{J}_{CP} , \quad (10)$$

where the  $\Delta m_{ij}^2$ 's are the light neutrino mass squared differences and  $\mathcal{J}_{CP}$  is the imaginary part of an invariant quartet appearing in the difference of the CP-conjugated neutrino oscillation probabilities  $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ . One can easily get:

$$\begin{aligned}\mathcal{J}_{CP} &\equiv \text{Im} [(U_\nu)_{11}(U_\nu)_{22}(U_\nu)_{12}^*(U_\nu)_{21}^*] \\ &= \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \cos(\theta_{13}) \sin \delta,\end{aligned}\quad (11)$$

where the  $\theta_{ij}$  are the mixing angles appearing in the standard parametrization adopted in [12]. Alternatively, one can use Eq. (10) and write:

$$\mathcal{J}_{CP} = -\frac{\text{Im} [(\mathcal{H}_\nu)_{12}(\mathcal{H}_\nu)_{23}(\mathcal{H}_\nu)_{31}]}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}.\quad (12)$$

This expression again has the advantage of allowing the computation of the leptonic low-energy CP rephasing invariant  $\mathcal{J}_{CP}$ , without resorting to the mixing matrix  $U_\nu$ .

It is also possible to write WB invariants useful to leptogenesis [8] as well as WB invariant conditions for CP conservation in the leptonic sector relevant in specific frameworks [11], [13].

## 4 CP Asymmetries in Heavy Majorana Neutrino Decays

The starting point in leptogenesis scenarios is the  $CP$  asymmetry generated through the interference between tree-level and one-loop heavy Majorana neutrino decay diagrams. In the simplest extension of the SM, such diagrams correspond to the decay of the Majorana neutrino into a lepton and a Higgs boson. Considering the decay of one heavy Majorana neutrino  $N_j$ , this asymmetry is given by:

$$\varepsilon_j = \frac{\Gamma(N_j \rightarrow \ell \phi) - \Gamma(N_j \rightarrow \bar{\ell} \phi^\dagger)}{\Gamma(N_j \rightarrow \ell \phi) + \Gamma(N_j \rightarrow \bar{\ell} \phi^\dagger)}.\quad (13)$$

In terms of the Dirac neutrino Yukawa couplings the CP asymmetry (13) is [14]:

$$\varepsilon_j = \frac{1}{8\pi(Y_\nu^\dagger Y_\nu)_{jj}} \sum_{k \neq j} \text{Im}[(Y_\nu^\dagger Y_\nu)_{jk}^2] f\left(\frac{M_k^2}{M_j^2}\right),\quad (14)$$

where the index  $j$  is not summed over in  $(Y_\nu^\dagger Y_\nu)_{jj}$ . The loop function  $f(x)$  includes the one-loop vertex and self-energy corrections to the heavy neutrino decay amplitudes,

$$f(x) = \sqrt{x} \left[ (1+x) \ln\left(\frac{x}{1+x}\right) + \frac{2-x}{1-x} \right].\quad (15)$$

From Eq. (14) it can be readily seen that the CP asymmetries are only sensitive to the CP-violating phases appearing in  $Y_\nu^\dagger Y_\nu$  (or equivalently in  $m_D^\dagger m_D$ ) in the WB where  $M_R$  and  $m_\ell$  are diagonal.

Let us consider the hierarchical case  $M_1 < M_2 \ll M_3$ . In this case only the decay of the lightest heavy neutrino  $N_1$  is relevant for leptogenesis, provided the interactions of  $N_1$  are in thermal equilibrium at the time  $N_{2,3}$  decay, so that the asymmetries produced by the latter are

erased before  $N_1$  decays. In this situation, it is sufficient to take into account the CP asymmetry  $\varepsilon_1$ . Since in the limit  $x \gg 1$  the function  $f(x)$  can be approximated by<sup>3</sup>  $f(x) \simeq -3/(2\sqrt{x})$ , we have from Eq. (14)

$$\varepsilon_1 = -\frac{3}{16\pi(Y_\nu^\dagger Y_\nu)_{11}} \sum_{k=2,3} \text{Im}[(Y_\nu^\dagger Y_\nu)_{1k}^2] \frac{M_1}{M_k}, \quad (16)$$

which can be recast in the form [15]

$$\varepsilon_1 \simeq -\frac{3 M_1}{16 \pi} \frac{\text{Im} [Y_\nu^\dagger Y_\nu D_R^{-1} Y_\nu^T Y_\nu^*]_{11}}{(Y_\nu^\dagger Y_\nu)_{11}} = \frac{3 M_1}{16 \pi v^2} \frac{\text{Im} [Y_\nu^\dagger \mathcal{M}_\nu Y_\nu^*]_{11}}{(Y_\nu^\dagger Y_\nu)_{11}}, \quad (17)$$

using the seesaw relation given in Eq. (7).

## 5 On the Link between Leptogenesis and Low-Energy CP Violation

In this section we analyze the possible connection between CP violation at low energies, measurable for example through neutrino oscillations, and leptogenesis. Of particular interest are the following questions:

- If the strength of CP violation at low energies in neutrino oscillations is measured, what can one infer about the viability or non-viability of leptogenesis? In particular, can one have viable leptogenesis even if there is no CP violation at low energies (i.e. no Dirac and no Majorana phases at low energies)?
- From the sign of the BAU, can one predict the sign of the CP asymmetries at low energies, namely the sign of  $\mathcal{J}_{CP}$ ?

We will show that having an explicit parametrization of  $m_D$  (or equivalently of  $Y_\nu = m_D/v$ ) is crucial not only to determine which phases are responsible for leptogenesis and which ones are relevant for leptonic CP violation at low energies, but also to analyze the relationship between these two phenomena.

From the available neutrino oscillation data, one obtains some information on the effective neutrino mass matrix  $\mathcal{M}_\nu$  which can be decomposed in the following way:

$$U_\nu d_\nu U_\nu^T = \mathcal{M}_\nu \equiv L L^T, \quad L \equiv i m_D D_R^{-1/2}. \quad (18)$$

The extraction of  $L$  from  $\mathcal{M}_\nu$  suffers from an intrinsic ambiguity [16] in the sense that, given a particular solution  $L_0$  of Eq. (18), the matrix  $L = L_0 R$  will also satisfy this equation, provided that  $R$  is an arbitrary orthogonal complex matrix,  $R \in O(3, C)$ , i.e.  $R R^T = 1$ . It is useful to take as a reference solution  $L_0 \equiv U_\nu d_\nu^{1/2}$ , so that:

$$L \equiv U_\nu d_\nu^{1/2} R. \quad (19)$$

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<sup>3</sup>This approximation can be reasonably used for  $x \gtrsim 15$ .

Since three of the phases of  $m_D$  can be eliminated, the matrix  $L$  has 15 independent parameters. The parametrization of  $L$  given in Eq. (19) has the interesting feature that all its parameters are conveniently distributed among  $U_\nu$ ,  $d_\nu$  and  $R$ , which contain 6 (3 angles + 3 phases), 3 and 6 (3 angles + 3 phases) independent parameters, respectively. Of the 18 parameters present in the Lagrangian of the fundamental theory described by  $m_D$  and  $D_R$ , only 9 appear at low energy in  $\mathcal{M}_\nu$  through the seesaw mechanism. To further disentangle  $m_D$  from  $D_R$  in  $L$ , one needs the 3 remaining inputs, namely the three heavy Majorana masses of  $D_R$ .

Coming back to the connection between leptogenesis and low-energy data, it is important to note that  $U_\nu$  does not appear in the relevant combination for leptogenesis  $Y_\nu^\dagger Y_\nu$ , in the same way as  $R$  does not appear in  $\mathcal{M}_\nu$ . Indeed, one has:

$$m_D^\dagger m_D = D_R^{1/2} R^\dagger d_\nu R D_R^{1/2}. \quad (20)$$

From the above discussion, it follows that it is possible to write  $m_D$  in the form  $m_D = -i U_\nu d_\nu^{1/2} R D_R^{1/2}$  in such a way that leptogenesis and the low-energy neutrino data (contained in  $\mathcal{M}_\nu$ ) depend on two independent sets of CP-violating phases, respectively those in  $R$  and those in  $U_\nu$ . In particular, one may have viable leptogenesis even in the limit where there are no CP-violating phases (neither Dirac nor Majorana) in  $U_\nu$  and hence, no CP violation at low energies [9]. Therefore, in general it is not possible to establish a link between low-energy CP violation and leptogenesis. This connection is model dependent: it can be drawn only by specifying a particular *ansatz* for the fundamental parameters of the seesaw,  $m_D$  and  $D_R$ , as will be done in the following sections.

The relevance of the matrix  $R$  for leptogenesis can be rendered even more explicit [17] by rewriting the  $\varepsilon_1$  asymmetry by means of Eq. (20) and defining  $R_{ij} = |R_{ij}| e^{i\varphi_{ij}/2}$ ,  $\Delta m_\odot^2 \equiv \Delta m_{21}^2$  and  $\Delta m_\oplus^2 \equiv \Delta m_{32}^2$ . In the case of hierarchical heavy Majorana neutrinos, say  $M_1 \ll M_2 \ll M_3$  one obtains

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_1}{v^2} \frac{\Delta m_\oplus^2 |R_{31}|^2 \sin \varphi_{31} - \Delta m_\odot^2 |R_{11}|^2 \sin \varphi_{11}}{m_1 |R_{11}|^2 + m_2 |R_{21}|^2 + m_3 |R_{31}|^2}, \quad (21)$$

and we recover what one would have expected by intuition, namely that the physical quantities involved in determining  $\varepsilon_1$  are just  $M_1$ , the spectrum of the light neutrinos,  $m_i$ , and the first column of  $R$ , which expresses the composition of the lightest heavy Majorana neutrino in terms of the light neutrino masses  $m_i$ .

As stressed before, different *ansätze* for  $R$  have no direct impact on CP violation at low energy; the impact is in a sense indirect because  $R$  specifies if dominance of some heavy Majorana neutrino is at work in the seesaw mechanism [18].

In conclusion, the link between leptogenesis and low-energy CP violation can only be established in the framework of specific *ansätze* for the leptonic mass terms of the Lagrangian. In order to derive a necessary condition for such a link to exist, it is convenient to use the following triangular parametrization for  $m_D$ :

### Triangular parametrization

It can be easily shown that any arbitrary complex matrix can be written as the product of a unitary matrix  $U$  with a lower triangular matrix  $Y_\Delta$ . In particular, the Dirac neutrino mass

matrix can be written as:

$$m_D = v U Y_\Delta, \quad (22)$$

with  $Y_\Delta$  of the form:

$$Y_\Delta = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} e^{i\phi_{21}} & y_{22} & 0 \\ y_{31} e^{i\phi_{31}} & y_{32} e^{i\phi_{32}} & y_{33} \end{pmatrix}, \quad (23)$$

where  $y_{ij}$  are real positive numbers. Since  $U$  is unitary, in general it contains six phases. However, three of these phases can be rephased away by a simultaneous phase transformation on  $\nu_L^0$ ,  $\ell_L^0$ , which leaves the leptonic charged current invariant. Under this transformation,  $m_D \rightarrow P_\xi m_D$ , with  $P_\xi = \text{diag}(e^{i\xi_1}, e^{i\xi_2}, e^{i\xi_3})$ . Furthermore,  $Y_\Delta$  defined in Eq. (23) can be written as:

$$Y_\Delta = P_\beta^\dagger \hat{Y}_\Delta P_\beta, \quad (24)$$

where  $P_\beta = \text{diag}(1, e^{i\beta_1}, e^{i\beta_2})$  with  $\beta_1 = -\phi_{21}$ ,  $\beta_2 = -\phi_{31}$  and

$$\hat{Y}_\Delta = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} & y_{22} & 0 \\ y_{31} & y_{32} e^{i\sigma} & y_{33} \end{pmatrix}, \quad (25)$$

with  $\sigma = \phi_{32} - \phi_{31} + \phi_{21}$ . It follows then from Eqs. (22) and (24) that the matrix  $m_D$  can be decomposed in the form

$$m_D = v U_\rho P_\alpha \hat{Y}_\Delta P_\beta, \quad (26)$$

where  $P_\alpha = \text{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$  and  $U_\rho$  is a unitary matrix containing only one phase  $\rho$ . Therefore, in the WB where  $m_\ell$  and  $M_R$  are diagonal and real, the phases  $\rho$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\sigma$ ,  $\beta_1$  and  $\beta_2$  are the only physical phases characterizing CP violation in the leptonic sector.

## A necessary condition

The phases relevant for leptogenesis are those contained in  $m_D^\dagger m_D$ . From Eqs. (24)-(26) we conclude that these phases are  $\sigma$ ,  $\beta_1$  and  $\beta_2$ , which are linear combinations of the phases  $\phi_{ij}$ . On the other hand, all the six phases of  $m_D$  contribute to the three phases of the effective neutrino mass matrix at low energies [8] which in turn controls CP violation in neutrino oscillations. Since the phases  $\alpha_1$ ,  $\alpha_2$  and  $\rho$  do not contribute to leptogenesis, it is clear that **a necessary condition**[19] for a direct link between leptogenesis and low-energy CP violation to exist is the requirement that the matrix  $U$  in Eq. (22) contains no CP-violating phases. Note that, although the above condition was derived in a specific WB and using the parametrization of Eq. (22), it can be applied to any model. This is due to the fact that starting from arbitrary leptonic mass matrices, one can always make WB transformations to render  $m_\ell$  and  $M_R$  diagonal, while  $m_D$  has the form of Eq. (22). A specific class of models which satisfy the above necessary

condition in a trivial way are those for which  $U = 1$ , leading to  $m_D = v Y_\Delta$ . This condition is necessary but not sufficient to allow for a prediction of the sign of the CP asymmetry in neutrino oscillations, given the observed sign of the BAU together with the low-energy data. More restrictive class of matrices  $m_D$  in triangular form have been considered and it was shown that, in an appropriate limiting case, these structures for  $m_D$  lead to the ones assumed by Frampton, Glashow and Yanagida in [20]. Let us consider the following form for  $m_D$ :

$$m_D = v Y_\Delta = v \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} e^{i\phi_{21}} & y_{22} & 0 \\ y_{31} e^{i\phi_{31}} & y_{32} e^{i\phi_{32}} & y_{33} \end{pmatrix} \quad (27)$$

Then, from Eq. (14) the CP asymmetry generated in the decay of the heavy Majorana neutrino  $N_j$  is

$$\varepsilon_j = -\frac{1}{8\pi(H_\Delta)_{jj}} \sum_{i \neq j} \text{Im}[(H_\Delta)_{ij}^2] f_{ij}, \quad (28)$$

where

$$H_\Delta = Y_\Delta^\dagger Y_\Delta \quad , \quad f_{ij} = f\left(\frac{M_i^2}{M_j^2}\right), \quad (29)$$

with  $f(x)$  defined in Eq. (15).

From Eqs. (27) and (29) we readily obtain

$$\begin{aligned} \text{Im}[(H_\Delta)_{21}^2] &= y_{21}^2 y_{22}^2 \sin(2\phi_{21}) + 2 y_{21} y_{22} y_{31} y_{32} \sin \theta_1 + y_{31}^2 y_{32}^2 \sin \theta_2, \\ \text{Im}[(H_\Delta)_{31}^2] &= y_{31}^2 y_{33}^2 \sin(2\phi_{31}), \\ \text{Im}[(H_\Delta)_{32}^2] &= y_{32}^2 y_{33}^2 \sin(2\phi_{32}), \end{aligned} \quad (30)$$

with  $\theta_1 = \phi_{21} + \phi_{31} - \phi_{32}$  and  $\theta_2 = 2(\phi_{31} - \phi_{32})$ .

All the information about light neutrino masses and mixing is fully contained in the effective neutrino mass matrix  $\mathcal{M}_\nu$  which is determined through the seesaw formula given by Eq. (7). In this case

$$\mathcal{M}_\nu = \frac{v^2}{M_1} \begin{pmatrix} y_{11}^2 & y_{11}y_{21}e^{i\phi_{21}} & y_{11}y_{31}e^{i\phi_{31}} \\ y_{11}y_{21}e^{i\phi_{21}} & y_{21}^2 e^{2i\phi_{21}} + y_{22}^2 \frac{M_1}{M_2} & y_{21}y_{31}e^{i(\phi_{31}+\phi_{21})} + y_{22}y_{32}\frac{M_1}{M_2} e^{i\phi_{32}} \\ y_{11}y_{31}e^{i\phi_{31}} & y_{21}y_{31}e^{i(\phi_{31}+\phi_{21})} + y_{22}y_{32}\frac{M_1}{M_2} e^{i\phi_{32}} & y_{31}^2 e^{2i\phi_{31}} + y_{33}^2 \frac{M_1}{M_2} + y_{32}^2 \frac{M_1}{M_2} e^{2i\phi_{32}} \end{pmatrix} \quad (31)$$

It follows from Eqs. (28)-(31) that, in principle, one can obtain simultaneously viable values for the CP asymmetries  $\varepsilon_j$  and a phenomenologically acceptable effective neutrino mass matrix in order to reproduce the solar, atmospheric and reactor neutrino data. This can be achieved by consistently choosing the values of the free parameters  $y_{ij}$ ,  $M_i$  and  $\phi_{ij}$ . Yet, a closer look at Eqs. (28)-(30) shows that there are terms contributing to  $\varepsilon_j$  which vanish independently from the others. This means that a non-vanishing value of  $\varepsilon_j$  can be guaranteed even for simpler structures for  $Y_\nu$ , which can be obtained from  $Y_\Delta$  assuming additional zero entries in the lower triangle<sup>4</sup>. A systematic study of the various possible textures, with emphasis on the link between low energy data and leptogenesis has been done [19].

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<sup>4</sup>Notice however that the vanishing of diagonal elements in  $Y_\Delta$  would imply  $\det(m_D) = 0$  and consequently,  $\det(\mathcal{M}_\nu) = 0$ , leading to the existence of massless light neutrinos.



## 6 Concluding Remarks

The phenomenon of CP violation plays a crucial role both in Particle Physics and Cosmology. In the context of Particle Physics, the breaking of CP is closely connected to the least established sectors of the SM, namely the Higgs sector and the Yukawa sector. The study of CP violation can thus be a good ground for the search for New Physics [21]. It is natural to wonder whether the various manifestations of CP violation ( at low energies in the quark and lepton sectors and at high energy in the existence of BAU ) have all a common origin [22]. We have presented here a brief review of the role of CP violation in generating BAU through leptogenesis, with emphasis on a possible connection between CP violation responsible for leptogenesis and leptonic CP violation at low energies, measurable through neutrino oscillations. We have emphasized that such a connection is only possible within specific ansatzes for the flavour structure of the fundamental leptonic mass matrices. This essentially means that the problem of connecting CP violation present in leptogenesis to CP violation detectable in neutrino oscillations cannot be separable from the general flavour problem.

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